3.13 The Per-Unit System

Let us say that the measure unit of age is 80 years (this is the base unit). Then a person who is 40 years old is half of the base unit. So we say the person’s age is 0.5 per unit.

\[ \text{Age per unit} = \frac{\text{age in years}}{\text{age base in years}} \]

Thus the age per unit is dimensionless. In power systems there are four base quantities required to define a per unit system. These are: power, voltage, current and impedance. Given two of these, all others can be defined using these two. In general for any quantity:

\[ \frac{\text{actual quantity}}{\text{base value of quantity}} = \text{Quantity per unit} \]

And for power systems we have:

\[
\begin{align*}
S_{pu} &= \frac{S}{S_B} \\
V_{pu} &= \frac{V}{V_B} \\
I_{pu} &= \frac{I}{I_B} \\
Z_{pu} &= \frac{Z}{Z_B}
\end{align*}
\]

N.B. The value of the numerators above may be a complex number, however, the denominator (the base value) is a positive real number.

Normally we select a three-phase power base \((S_B\) or \(\text{MVA}_B\)) and a line-to-line voltage base \((V_B\) or \(\text{kV}_B\)). From these two the other bases can be computed using circuit laws thus:

\[
I_B = \frac{S_B \sqrt{3}}{3 V_B} = \frac{S_B}{\sqrt{3}V_B}
\]

and

\[
Z_B = \frac{V_B}{\sqrt{3}I_B}
\]

Other forms for \(Z_B\) follow from the above equations:

\[
Z_B = \frac{(V_B)^2}{S_B}
\]

\[
Z_B = \frac{(kV_B)^2}{\text{MVA}_B}
\]

Note that \(V_B\) is line to line voltage base! and \(I_B\) is line current base. Also, \(S_B\) is 3-phase power base. Thus in dealing with per-phase quantities, the \(3\) and \(\sqrt{3}\) need to be taken into account.
Per unit quantities obey the circuit laws, thus:

\[ S_{pu} = V_{pu} I_{pu}^* \]

and

\[ V_{pu} = Z_{pu} I_{pu} \]

In terms of phase quantities, the three phase complex power to a load is given by:

\[ S_{L(3p)} = 3V_p I_p^* \]

and the phase current is related to the phase voltage by the load impedance thus:

\[ I_p = \frac{V_p}{Z_p} \]

Using this value of phase current in the equation above we have (solving for impedance):

\[ Z_p = \frac{3|V_p|^2}{S_{L(3p)}^*} = \frac{|V_{L-L}|^2}{S_{L(3p)}^*} \]

Therefore the phase impedance in per-unit is given by:

\[ Z_{pu} = \frac{Z_p}{Z_B} = \frac{|V_{L-L}|^2}{S_{L(3p)}^* Z_B} \]

and since \( Z_B = \frac{V_B^2}{S_B} \) (see last page) the above expression becomes:

\[ Z_{pu} = \frac{Z_p}{Z_B} = \frac{|V_{L-L}|^2}{S_{L(3p)}^*} \frac{S_B}{|V_B|^2} \frac{S_{L(3p)}^*}{S_{L(3p)}} \]

or

\[ Z_{pu} = \left( \frac{V_{pu}^2}{S_{L(3p)}} \right) \]

### 3.14 Change of Base

Usually if none are specified, the pu values given are on nameplate ratings as base. For example a generator whose impedance is 0.4 pu and whose ratings are 110 MVA and 24 kV, then the base MVA is assumed 110 MVA and base line to line voltage is 24 kV. Often the base for the system is different from the base for each particular generator or transformer, hence it is important to be able to express the pu value in terms of different bases. This is derived below.

Let \( Z_{pu}^{old} \) be the per-unit impedance on the power base \( S_B^{old} \) and voltage base \( V_B^{old} \). We would like to find \( Z_{pu}^{new} \) on the new power base \( S_B^{new} \) and voltage base \( V_B^{new} \). Let \( Z_\Omega \) be the "ohmic" value of the impedance. Then we have:
\[ Z_{pu}^{\text{old}} = \frac{Z_B}{Z_{old}^{\text{old}}} = Z_\Omega \frac{S_B^{\text{old}}}{(V_B^{\text{old}})^2} \]

and similarly

\[ Z_{pu}^{\text{new}} = \frac{Z_B}{Z_{new}^{\text{new}}} = Z_\Omega \frac{S_B^{\text{new}}}{(V_B^{\text{new}})^2} \]

By comparison of these two expressions it is evident that:

\[ Z_{pu}^{\text{new}} = Z_{pu}^{\text{old}} \frac{S_B^{\text{new}}}{S_B^{\text{old}}} \left( \frac{V_B^{\text{old}}}{V_B^{\text{new}}} \right)^2 \]

This last expression is very useful in transforming per-unit values from one set of bases to another.

Example. A transformer has a voltage rating of 110/220 V and a power rating of 1000 VA. The impedance of the transformer on the low voltage side is 5 + j33. Find the per-unit impedance viewed from each side assuming the bases are equal to the voltages and power ratings on each side.

Solution using Matlab:

From the given quantities:

\[
\begin{align*}
V_{B1} &= 110; \quad V_{B2} = 220; \quad S_{B1} = 1000; \quad S_{B2} = 1000; \\
Z_1 &= 5 + j33; \quad Z_2 = 5 + j4; \\
Z_{B1} &= V_{B1}^2/S_{B1}; \quad Z_{B2} = V_{B2}^2/S_{B2}; \\
Z_{pu1} &= Z_1/Z_{B1}; \\
Z_{pu2} &= Z_2/Z_{B2}
\end{align*}
\]

\[
\begin{align*}
Z_{pu1} &= 0.4132 + 2.7273i \\
Z_{pu2} &= 0.4132 + 2.7273i
\end{align*}
\]

Thus, with proper selection of \( S_{\text{base}} \) and \( V_{\text{base}} \) on each side of a transformer, the per-unit impedance of the transformer becomes the same viewed from either side! Thus the transformer may be removed and replaced with its per-unit impedance (but we have to keep in mind that now there are two different values of voltage base, one on each side of the transformer).

**Rules for selection of voltage bases around a transformer:** Let the turns ratio of the transformer be \( \alpha = \frac{N_1}{N_2} \), and suppose there is a connection gain of \( k \). If this is a \( Y-Y \) or \( \Delta-\Delta \) connected transformer, then \( k = 1 \). If it is a \( Y-\Delta \) connection, then \( k = \sqrt{3} \). If it is \( \Delta-Y \) then \( k = 1/\sqrt{3} \). Select a voltage base on side 1, say \( V_{b1} \). Then the voltage base on side 2 must be: \( V_{b2} = k\alpha V_{b1} \). The power base, of course is the same everywhere in the power system.
Example 3.7

The one line diagram of a power system is shown in fig. 3.29 page 92. Select a common base of 100 MVA and 22 kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per unit. The manufacturer’s data for each device is given on page 92. The three phase load on bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactances of 48.4 Ω and 65.43 Ω, respectively.

Outline of solution: first find the voltage base in all sections of the system. In this case there are five sections. Note that in this case there is no mention of how the transformers are connected (i.e. Y or delta). Hence we consider only the turns ratio (otherwise there would be some \( \sqrt{3} \) factors as well).

The Matlab program follows:

```matlab
VLL = 10.45;
VB1 = 22; VB2 = 22*220/22; VB4 = 220*11/220; VB5 = 22*110/22;
Xg = 0.18*100/90
Xt1 = 0.1*100/50
Xt2 = 0.06*100/40
Xt3 = 0.064*100/40
Xt4 = 0.08*100/40
Xm = 0.185*(100/66.5)*(10.45/11)^2
ZB2 = (220)^2/100
ZB5 = (110)^2/100
Xline1 = 48.4/484
Xline2 = 65.43/121
SL = 57*(0.6 + j*sin(acos(0.6)))
ZL = VLL^2/conj(SL)
ZB4 = (11)^2/100
ZLpu = ZL/ZB4

Xg =
  0.2000
Xt1 =
  0.2000
Xt2 =
  0.1500
Xt3 =
  0.1600
Xt4 =
  0.2000
Xm =
  0.2511
ZB2 =
  484
ZB5 =
  121
Xline1 =
  0.1000
Xline2 =
Note these are then incorporated into the impedance diagram on page 94. Note there are no transformers showing! Just an elementary one phase diagram with impedances. To go back to actual values, we need to know the MVA_B and the V_B at every point in the system. As stated earlier, the MVA_B is the same everywhere, but the V_B changes as we go from one side of a transformer to the other.

Example 3.8

The motor of example 3.7 operates at full load 0.8 power factor leading at a terminal voltage of 10.47 kV.
(a) Determine the voltage at the generator bus.
(b) Determine the generator and motor emfs.

The solution is found using ordinary circuit analysis. Of course, the answers will be "pu" all the way!

The Matlab program follows:

```matlab
ZL = 0.95 + j*1.2667;
V4=10.45/11
Sm = 66.5/100*(0.8 - j*.6)
Im = conj(Sm)/conj(V4)
IL = V4/ZL
I = Im + IL
X11 = 0.45*0.9/(0.45+0.9);
V1 = V4+j*X11*I; V1M=abs(V1), V1ang=angle(V1)*180/pi
Eg = V1 + j*0.2*I; EgM=abs(Eg), Egang=angle(Eg)*180/pi
Em = V4 - j*0.25*Im; EmM=abs(Em), Emang=angle(Em)*180/pi

V4 =
    0.9500
Sm =
    0.5320 - 0.3990i
Im =
    0.5600 + 0.4200i
IL =
    0.3600 - 0.4800i
I =
    0.9200 - 0.0600i
V1M =
```

0.5407
SL =
    34.2000 +45.6000i
ZL =
    1.1495 + 1.5327i
ZB4 =
    1.2100
ZLpu =
    0.9500 + 1.2667i
1.0066  
\text{Vlang} =  
15.9139  
\text{EgM} =  
1.0826  
\text{Egang} =  
25.1445  
\text{EmM} =  
1.0642  
\text{Emang} =  
-7.5591